เฉลยพิชิตข้อสอบ PAT 1
เรื่อง ฟังก์ชันตรีโกณมิติ (PART I)

ข้อ 5 <u>ตอบ</u> **180**

$* \quad \sin^2 1^\circ + \sin^2 2^\circ + \ldots\ldots + \sin^2 88^\circ + \sin^2 89^\circ$

$= (\sin^2 1^\circ + \sin^2 89^\circ) + (\sin^2 2^\circ + \sin^2 88^\circ) + \ldots\ldots + (\sin^2 44^\circ + \sin^2 46^\circ) + \sin^2 45^\circ$

$= (\sin^2 1^\circ + \cos^2 1^\circ) + (\sin^2 2^\circ + \cos^2 2^\circ) + \ldots\ldots + (\sin^2 44^\circ + \cos^2 44^\circ) + \left(\frac{1}{\sqrt{2}}\right)^2$

$= 1 + 1 + 1 + \ldots\ldots_{44 \text{ ตัว}} + 1 + \frac{1}{2} = 44.5$

$* \quad \sin^2 91^\circ + \sin^2 92^\circ + \ldots\ldots + \sin^2 179^\circ = \sin^2 89^\circ + \sin^2 88^\circ + \ldots\ldots + \sin^2 1^\circ = 44.5$

$* \quad \sin^2 181^\circ + \sin^2 182^\circ + \ldots\ldots + \sin^2 269^\circ = \sin^2 1^\circ + \sin^2 2^\circ + \ldots\ldots + \sin^2 89^\circ = 44.5$

$* \quad \sin^2 271^\circ + \sin^2 272^\circ + \ldots\ldots + \sin^2 359^\circ = \sin^2 89^\circ + \sin^2 88^\circ + \ldots\ldots + \sin^2 1^\circ = 44.5$

$\therefore \quad \sin^2 1^\circ + \sin^2 2^\circ + \sin^2 3^\circ + \ldots\ldots + \sin^2 360^\circ = 44.5 + \sin^2 90^\circ + 44.5 + \sin^2 180^\circ$

$+ 44.5 + \sin^2 270^\circ + 44.5 + \sin^2 360^\circ$

$= 178 + 1 + 0 + 1 + 0 = 180$

ข้อ 6 <u>ตอบ</u> 8100

$\sin^2 1^\circ + 2\sin^2 2^\circ + 3\sin^2 3^\circ + \ldots\ldots + 178\sin^2 178^\circ + 179\sin^2 179^\circ + 180\sin^2 180^\circ$

$\sin^2 1^\circ + 2\sin^2 2^\circ + 3\sin^2 3^\circ + \ldots\ldots + 178\sin^2 2^\circ + 179\sin^2 1^\circ + 0$

$180(\sin^2 1^\circ + \sin^2 2^\circ + \ldots\ldots + \sin^2 89^\circ) + 90\sin^2 90^\circ$

$180(44.5) + 90 = 90[2(44.5) + 1] = 90(90) = 8100$

ข้อ 9 หน้า 3.5

$\sin a + 7\sin b = 4\sin c + 8\sin d \rightarrow \sin a - 8\sin d = 4\sin c - 7\sin b$ ———(1)

$\cos a + 7\cos b = 4\cos c + 8\cos d \rightarrow \cos a - 8\cos d = 4\cos c - 7\cos b$ ———(2)

$(1)^2, \ \sin^2 a - 16\sin a \sin d + 64\sin^2 d = 16\sin^2 c - 56\sin b \sin c + 49\sin^2 b$ ———(3)

$(2)^2, \ \cos^2 a - 16\cos a \cos d + 64\cos^2 d = 16\cos^2 c - 56\cos b \cos c + 49\cos^2 b$ ———(4)

$(3) + (4), \ 65 - 16\cos(a - d) = 65 - 56\cos(b - c)$

$\therefore \quad \frac{\cos(a-d)}{\cos(b-c)} = \frac{56}{16} = \frac{7}{2} = 3.5$

ข้อ 10 **ตอบ** 1

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\frac{\sin(A-B)}{\cos A\cos B} = \frac{\sin A\cos B}{\cos A\cos B} - \frac{\cos A\sin B}{\cos A\cos B}$$

$$\frac{\sin(A-B)}{\cos A\cos B} = \tan A - \tan B$$

$$\frac{1}{\cos A\cos B} = \frac{\tan A - \tan B}{\sin(A-B)}$$

$$\frac{1}{\cos 1^\circ\cos 0^\circ} + \frac{1}{\cos 2^\circ\cos 1^\circ} + \frac{1}{\cos 3^\circ\cos 2^\circ} + \ldots\ldots + \frac{1}{\cos 45^\circ\cos 44^\circ} = \frac{1}{\sin n^\circ}$$

$$\frac{\tan 1^\circ - \tan 0^\circ}{\sin(1^\circ - 0^\circ)} + \frac{\tan 2^\circ - \tan 1^\circ}{\sin(2^\circ - 1^\circ)} + \frac{\tan 3^\circ - \tan 2^\circ}{\sin(3^\circ - 2^\circ)} + \ldots\ldots + \frac{\tan 45^\circ - \tan 44^\circ}{\sin(45^\circ - 44^\circ)} = \frac{1}{\sin n^\circ}$$

$$\frac{1}{\sin 1^\circ}(\tan 1^\circ - \tan 0^\circ + \tan 2^\circ - \tan 1^\circ + \tan 3^\circ - \tan 2^\circ + \ldots\ldots + \tan 45^\circ - \tan 44^\circ) = \frac{1}{\sin n^\circ}$$

$$\frac{1}{\sin 1^\circ}(\tan 45^\circ - \tan 0^\circ) = \frac{1}{\sin n^\circ} \rightarrow \frac{1}{\sin 1^\circ} = \frac{1}{\sin n^\circ} \quad \therefore n = 1$$

ข้อ 23 **ตอบ** 0

โจทย์กำหนด A, B เป็นมุมแหลม (acute = แหลม)

$$3\sin^2 A + 2\sin^2 B = 1 \rightarrow 3\sin^2 A = 1 - 2\sin^2 B \rightarrow 3\sin^2 A = \cos 2B$$

$$3\sin 2A - 2\sin 2B = 0 \rightarrow \sin 2B = \frac{3}{2}\sin 2A$$

$$\cos(A+2B) = \cos A\cos 2B - \sin A\sin 2B$$

$$= \cos A(3\sin^2 A) - \sin A(\frac{3}{2}\sin 2A)$$

$$= 3\sin^2 A\cos A - \frac{3}{2}\sin A(2\sin A\cos A) = 0$$

ข้อ 25 **ตอบ** 9

$$(3 - 4\sin^2 9^\circ)(3 - 4\sin^2 27^\circ) = \cot n^\circ$$

$$\left(\frac{3\sin 9^\circ - 4\sin^3 9^\circ}{\sin 9^\circ}\right)\left(\frac{3\sin 27^\circ - 4\sin^3 27^\circ}{\sin 27^\circ}\right) = \cot n^\circ$$

$$\left(\frac{\sin 27^\circ}{\sin 9^\circ}\right)\left(\frac{\sin 81^\circ}{\sin 27^\circ}\right) = \cot n^\circ \rightarrow \frac{\cos 9^\circ}{\sin 9^\circ} = \cot n^\circ$$

$$\cot 9^\circ = \cot n^\circ \quad \therefore n = 9$$

ข้อ 26 <u>ตอบ</u> 1

$\cot 9^\circ[4(1-\cos^2 9^\circ)-1][4(1-\sin^2 63^\circ)-1]$

$\cot 9^\circ[3-4\cos^2 9^\circ][3-4\sin^2 63^\circ]$

$\cot 9^\circ[-(4\cos^2 9^\circ-3)][-(4\cos^2 27^\circ-3)]$

$\frac{\cos 9^\circ}{\sin 9^\circ}(4\cos^2 9^\circ-3)(4\cos^2 27^\circ-3)$

$\frac{\cos 9^\circ}{\sin 9^\circ}\left(\frac{4\cos^3 9^\circ-3\cos 9^\circ}{\cos 9^\circ}\right)\left(\frac{4\cos^3 27^\circ-3\cos 27^\circ}{\cos 27^\circ}\right)$

$\frac{1}{\sin 9^\circ}(\cos 27^\circ)\left(\frac{\cos 81^\circ}{\cos 27^\circ}\right) = \left(\frac{1}{\sin 9^\circ}\right)(\sin 9^\circ) = 1$

ข้อ 27 <u>ตอบ</u> b

$\tan 60^\circ = \tan 3(20^\circ) \rightarrow \sqrt{3} = \frac{3\tan 20^\circ-\tan^3 20^\circ}{1-3\tan^2 20^\circ} \rightarrow \sqrt{3}(1-3\tan^2 20^\circ) = 3\tan 20^\circ-\tan^3 20^\circ$

$[\sqrt{3}(1-3\tan^2 20^\circ)]^2 = (3\tan 20^\circ-\tan^3 20^\circ)^2 \rightarrow 3(1-3\tan^2 20^\circ)^2 = 9\tan^2 20^\circ-6\tan^4 20^\circ+\tan^6 20^\circ$

$3(1-6\tan^2 20^\circ+9\tan^4 20^\circ) = 9\tan^2 20^\circ-6\tan^4 20^\circ+\tan^6 20^\circ$

$3-18\tan^2 20^\circ+27\tan^4 20^\circ = 9\tan^2 20^\circ-6\tan^4 20^\circ+\tan^6 20^\circ$

$\tan^6 20^\circ-33\tan^4 20^\circ+27\tan^2 20^\circ = 3$

ข้อ 28 <u>ตอบ</u> 2

$\sin A\sin(60^\circ-A)\sin(60^\circ+A)$

$\sin A[\sin 60^\circ\cos A-\cos 60^\circ\sin A][\sin 60^\circ\cos A+\cos 60^\circ\sin A]$

$\sin A[\frac{\sqrt{3}}{2}\cos A-\frac{1}{2}\sin A][\frac{\sqrt{3}}{2}\cos A+\frac{1}{2}\sin A]$

$\sin A[\frac{3}{4}\cos^2 A-\frac{1}{4}\sin^2 A]$

$\frac{1}{4}\sin A[3(1-\sin^2 A)-\sin^2 A] = \frac{1}{4}\sin A[3-4\sin^2 A] = \frac{1}{4}[3\sin A-4\sin^3 A]$

$= \frac{1}{4}\sin 3A$

ข้อ 29 <u>ตอบ</u> $\frac{1}{6}$

$\sin 3A = \frac{2}{3}$

$\therefore \sin A\sin(60^\circ-A)\sin(60^\circ+A) = \frac{1}{4}\sin 3A$ (จากข้อ 28)

$= \frac{1}{4}\left(\frac{2}{3}\right) = \frac{1}{6}$

ข้อ 30 **ตอบ** 2

$$\sin 25^\circ \sin 85^\circ \sin 35^\circ = \sin 25^\circ \sin(60^\circ - 25^\circ)\sin(60^\circ + 25^\circ)$$

$$= \frac{1}{4}\sin 3(25^\circ) \quad \text{จากข้อ 28}$$

$$= \frac{1}{4}\sin 75^\circ$$

$$\therefore \frac{\sin 25^\circ \sin 85^\circ \sin 35^\circ}{\sin 75^\circ} = \frac{\frac{1}{4}\sin 75^\circ}{\sin 75^\circ} = \frac{1}{4} = \frac{1}{2}\sin 30^\circ = \frac{1}{2}(2\sin 15^\circ \cos 15^\circ)$$

$$= \sin 15^\circ \cos 15^\circ = \sin 15^\circ \sin 75^\circ$$

ข้อ 31 **ตอบ** 255

$\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{a}{b}$ เมื่อ $\frac{a}{b}$ เป็นเศษส่วนอย่างต่ำ

$$(\sin 70^\circ \sin 50^\circ \sin 10^\circ)\frac{1}{2} = \frac{a}{b}$$

$$\sin 10^\circ \sin(60^\circ - 10^\circ)\sin(60^\circ + 10^\circ)\left(\frac{1}{2}\right) = \frac{a}{b}$$

$$\left(\frac{1}{4}\sin 30^\circ\right)\left(\frac{1}{2}\right) = \frac{a}{b} \rightarrow \frac{a}{b} = \frac{1}{4}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{16} \rightarrow a = 1,\ b = 16$$

$$\therefore b^2 - a = 16^2 - 1 = 256 - 1 = 255$$

ข้อ 32 **ตอบ** 1

$$\tan 5^\circ \tan 55^\circ \tan 65^\circ \tan 75^\circ$$

$$\frac{\sin 5^\circ \sin 55^\circ \sin 65^\circ}{\cos 5^\circ \cos 55^\circ \cos 65^\circ}\tan 75^\circ$$

$$\frac{\sin 5^\circ \sin 55^\circ \sin 65^\circ}{\sin 85^\circ \sin 35^\circ \sin 25^\circ}\tan 75^\circ$$

$$\frac{\sin 5^\circ \sin(60^\circ - 5^\circ)\sin(60^\circ + 5^\circ)}{\sin 25^\circ \sin(60^\circ - 25^\circ)\sin(60^\circ + 25^\circ)}\tan 75^\circ$$

$$\frac{\frac{1}{4}\sin 15^\circ}{\frac{1}{4}\sin 75^\circ}\tan 75^\circ = \frac{\cos 75^\circ}{\sin 75^\circ}\tan 75^\circ = \cot 75^\circ \tan 75^\circ = 1$$

ขอให้ทุกคนโชคดีในการสอบนะครับ

ด้วยความห่วงใยจากใจจริง

พี่ช้าง